Technical Notes

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Unsteady Vortical Disturbances Around a Thin Airfoil in the Presence of a Wall

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I. Introduction

THIS Note investigates incompressible flows with unsteady upstream vorticity disturbances around a thin-plate airfoil in the presence of a wall. One objective of the paper is to determine the effect of the solid wall on the unsteady lift acting on the airfoil. Such fluctuating aerodynamic forces occur in a variety of engineering problems, such as wings during takeoff and landing, structures in ground transportation or on top of buildings that are subject to atmospheric turbulence, and ships in restricted waters. Numerous studies[1-4] have been carried out to determine the ground effect on steady flows. The results indicate a significant increase in the lift as the ratio of the airfoil distance to the wall to its chord length becomes of order one or smaller. This suggests that, under the same condition and at least at low reduced frequency, the presence of the wall will cause an equally significant increase in the value of the fluctuating lift.

Another objective of the present Note is to determine the unsteady velocity field downstream of the airfoil. This velocity field depends on the strength of vortex shedding in response to upstream nonuniformities, and thus it is directly related by the Kutta condition at the airfoil trailing edge to the fluctuating circulation around the airfoil. Interest in such velocity fields arises in recent studies[5-7] of the mechanism of turbulent drag reduction by large eddy-breakup devices.

The problem of a gust convected by an incompressible inviscid flow around an airfoil was first studied by Sears[8] who derived an expression for the unsteady lift known as the Sears function which is now extensively used in noise and forced vibration analysis. The present Note can be considered as an extension of Sears' work to include the effect of a wall located beneath the airfoil.

II. Mathematical Formulation

Figure 1 shows a schematic of a thin-plate airfoil of chord length c in two-dimensional, inviscid, incompressible flow at a distance h above an infinitely long, flat wall. The coordinate system $\boldsymbol{x} = \{x_1,x_2\}$ is such that the x_1 direction is taken parallel to the wall and the x_2 direction is perpendicular to the wall. The origin is located at the wall so that the x_2 axis passes by the airfoil midchord. Let $\boldsymbol{i}_1$ and $\boldsymbol{i}_2$ be the unit vectors in the x_1 and x_2 directions, respectively. Because viscosity is neglected, the mean velocity U of the flow is uniform in the x_1 direction. Imposed on the flow is a periodic disturbance convected by the mean flow of amplitude ϵu, where $\epsilon \ll 1$. All lengths will be normalized with respect to $c/2$, the time t with respect to $c/2U$, the velocities with respect to U, and the unsteady pressure p' is nondimensionalized with respect to $\epsilon \rho_0 U^2$, where ρ_0 is the density of the fluid. The total velocity field then can be written as

$$\boldsymbol{V} = \boldsymbol{i}_1 + \epsilon \boldsymbol{u}(x_1,x_2,t) \tag{1}$$

The perturbation velocity field $\boldsymbol{u} = \{u_1,u_2\}$ has magnitude of order of unity and, because the vortical disturbance is convected by the mean flow, the expression for $\boldsymbol{u}$ far upstream of the airfoil can be Fourier-decomposed so that

$$\boldsymbol{u}_\infty(x_1,x_2,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \boldsymbol{a}(\boldsymbol{k}) \exp[j(k_1 t - \boldsymbol{k}\cdot\boldsymbol{x})]\, \mathrm{d}k_1 \mathrm{d}k_2 \tag{2}$$

where

$$\boldsymbol{k} = k_1\boldsymbol{i}_1 + k_2\boldsymbol{i}_2 \tag{3}$$

and where k_1 is the usual reduced frequency and $j = \sqrt{-1}$. The function $\boldsymbol{a} = \{a_1,a_2\}$ is the Fourier transform of the upstream disturbance $\boldsymbol{u}_\infty$. The continuity equation requires that

$$\boldsymbol{a}\cdot\boldsymbol{k} = 0 \tag{4}$$

Although we allow slip along the wall, the normal velocity along the surface of the wall ($x_2 = 0$) is zero. Therefore,

$$a_1(k_1,-k_2) = a_1(k_1,k_2), \qquad a_2(k_1,-k_2) = -a_2(k_1,k_2) \tag{5}$$

As a result, Eq. (2) can be written as

$$\boldsymbol{u}_\infty(x_1,x_2,t) = 2\int_{-\infty}^{\infty}\int_{0}^{\infty} [\boldsymbol{i}_1 a_1 \cos(k_2x_2) - \boldsymbol{i}_2 j a_2 \sin(k_2x_2)] \times \exp[jk_1(t - x_1)]\, \mathrm{d}k_2\, \mathrm{d}k_1 \tag{6}$$

It is convenient to define $k = |\boldsymbol{k}|$ and $a^2(k_1,k_2) = |\boldsymbol{a}|^2$, hence,

$$a_1 = \frac{k_2}{k} a(k_1k_2) \qquad a_2 = \frac{-k_1}{k} a(k_1,k_2) \tag{7}$$

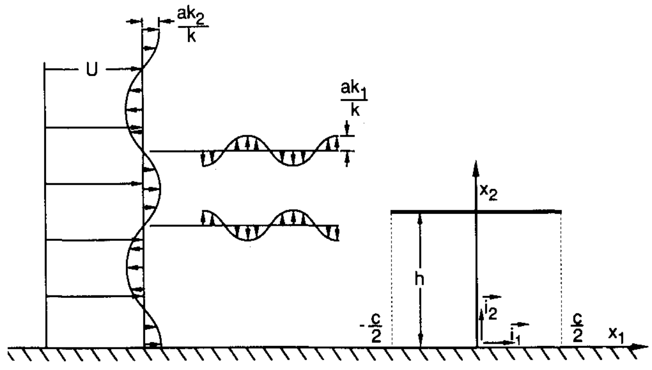

Fig. 1 Schematic of a thin airfoil in a gust in the presence of a wall.

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Without loss of generality, we can consider a single harmonic component of the upstream disturbance. Thus,

$$u_\infty = 2a(k_1,k_2)\left[i_1\frac{k_2}{k}\cos(k_2x_2) + i_2\frac{jk_1}{k}\sin(k_2x_2)\right]$$
$$\times \exp[jk_1(t - x_1)] \tag{8}$$

The total unsteady velocity then can be written

$$u = u^{(1)} + u_\infty \tag{9}$$

where $u^{(1)}$ is irrotational and solenoidal. The impermeability condition along the airfoil is expressed as

$$u_2^{(1)} = -u_{2\infty} \tag{10}$$

which, using Eq. (8), gives

$$u_2^{(1)} = -2a(k_1,k_2)\frac{jk_1}{k}\sin(k_2h)\exp[jk_1(t - x_1)], \tag{11}$$

for $-1 < x_1 < 1$

At large distance from the airfoil,

$$u^{(1)} \to 0, \text{ as } x_1^2 + x_2^2 \to \infty \tag{12}$$

Finally, since the pressure is continuous in the wake, the velocity jump $\Delta u_1^{(1)}$ across the wake must satisfy

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x_1}\right)\Delta u_1^{(1)}(x_1,t) = 0, \qquad \text{for } x_1 \geq 1 \tag{13}$$

which can be readily integrated to give

$$\Delta u_1^{(1)}(x_1,t) = C_0' \exp[jk_1(t - x_1 + 1)], \qquad \text{for } x_1 \geq 1 \tag{14}$$

where $C_0' e^{jk_1t}$ is the jump in the velocity at the trailing edge of the airfoil, and $\Delta f = f(x_1, h+0,t) - f(x_1,h-0,t)$ is the jump of the function across the line $x_2 = h$.

The boundary value problem for $u^{(1)}$ is now solved using the theory of sectionally analytic functions and the method of images. Using the first Plemelj formula[9] and noting that $\Delta u_1^{(2)}(x_1,h,t) = 0$, we can write

$$u_1^{(1)} - iu_2^{(1)} = \frac{1}{2\pi i}\int_{-1}^{\infty}\frac{\Delta u_1^{(1)}(x',t)}{x' - (z - ih)}\,dx'$$
$$-\frac{1}{2\pi i}\int_{-1}^{\infty}\frac{\Delta u_1^{(1)}(x',t)}{x' - (z + ih)}\,dx' \tag{15}$$

where $z = x_1 + ix_2$. Note that j is the unit imaginary number associated with the time phase, whereas i is the unit imaginary number associated with the x_2 direction. Using the second Plemlj formula at the airfoil surface, we obtain

$$(u_1^{(1)} - iu_2^{(1)})_+ + (u_1^{(1)} - iu_2^{(1)})_- = \frac{1}{\pi i}\left[\oint_{-1}^{\infty}\frac{\Delta u_1^{(1)}(x',t)}{x' - x_1}\,dx'\right.$$
$$\left. - \int_{-1}^{\infty}\frac{\Delta u_1^{(1)}(x',t)}{x' - x_1 - 2ih}\,dx'\right] \tag{16}$$

where $\pm$ represents the limiting values for $u_1^{(1)} - iu_2^{(1)}$ at $x_2 = h \pm 0$. Substituting Eq. (14) into Eq. (16) and taking the imaginary part with respect to i of Eq. (16), we obtain, after arrangement, the following singular integral equation:

$$\oint_{-1}^{1}\Delta u(x')\left[\frac{1}{x' - x_1} - \frac{x' - x_1}{(x' - x_1)^2 + 4h^2}\right]dx' = -e^{-jk_1x_1}$$
$$-C_0e^{jk_1}\int_1^{\infty}e^{-jk_1x'}\left[\frac{1}{x' - x_1} - \frac{x' - x_1}{(x' - x_1)^2 + 4h^2}\right]dx'$$

for $-1 < x_1 < 1$ (17)

where we have put

$$X = 4\pi a(k_1,k_2)\frac{jk_1}{k}\sin(k_2h)\, e^{jk_1t} \tag{18}$$

$$C_0 = \frac{C'}{X}e^{jk_1t} \tag{19}$$

$$\Delta u = \frac{\Delta u_1^{(1)}}{X} \tag{20}$$

The constant C_0 is determined by Kelvin's theorem[10] of conservation of circulation,

$$\int_{-1}^{1}\Delta u(x')\,dx' + C_0\int_1^{\infty}\exp[jk_1(1 - x')]\,dx' = 0 \tag{21}$$

which reduces to

$$\int_{-1}^{1}\Delta u(x')\,dx' - \frac{jC_0}{k_1} = 0 \tag{22}$$

assuming that k_1 has a small negative imaginary part. Finally, Δu must satisfy the Kutta condition

$$\Delta u(1) = 0 \tag{23}$$

Note that although the upstream disturbance, Eq. (8), depends on x_2 and k_2, by factoring out X, the integral equation (17) is independent of these parameters. This is due to the assumption of a flat-plate airfoil, which implies a uniform mean flow convecting with no distortion the vorticity disturbances.

As in the Sears problem, Δu has a square root singularity at the leading edge. It is therefore convenient to introduce the transformation $x' = -\cos\phi'$ and $x_1 = -\cos\phi_1$, so that the function $\Delta u \sin\phi'$ in the numerator of Eq. (17) becomes regular. Equation (17) then is solved numerically by collocation. Only about 40 collocation points are needed to get 1% accuracy in evaluating the singular Cauchy integral.

The pressure jump across the airfoil is easily found by integrating Euler's momentum equation with respect to x_1, so that

$$\Delta p(x_1) = -[\Delta u(x_1) - C_0] - jk_1\int_1^{x_1}\Delta u(x')\,dx', \tag{24}$$

for $-1 < x_1 < 1$

where $\Delta p = \Delta p'/X$. Finally, the unsteady lift on the airfoil for every Fourier component (8), is given by

$$L' = X(k_1,k_2,h,t)L_1'(k_1,h) \tag{25}$$

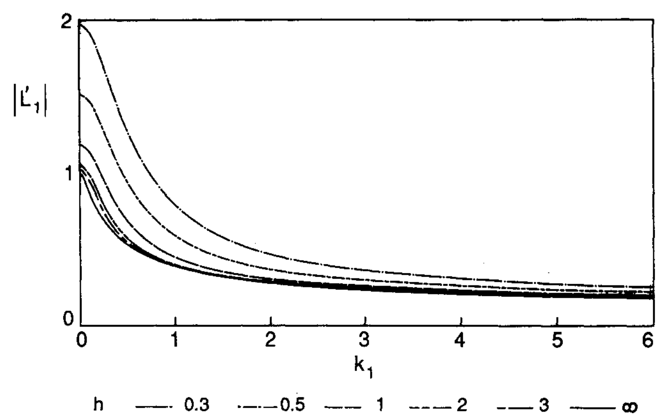

Fig. 2 Variation of the magnitude of the function L_1' vs the reduced frequency k_1 at different wall distance h.

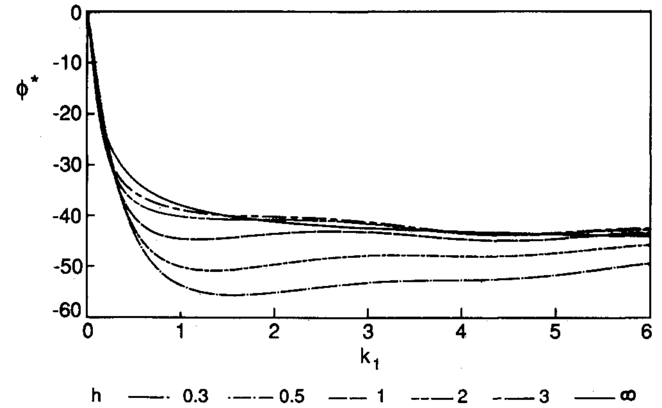

Fig. 3 Phase Φ^* of L_1' $\exp(-jk_1)$ vs k_1 at different wall distance h.

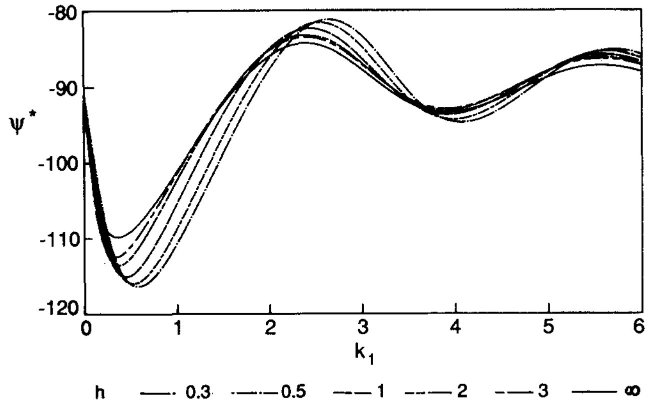

Fig. 5 Phase Ψ^* of C_0 $\exp(jk_1)$ vs k_1 at different wall distance h.

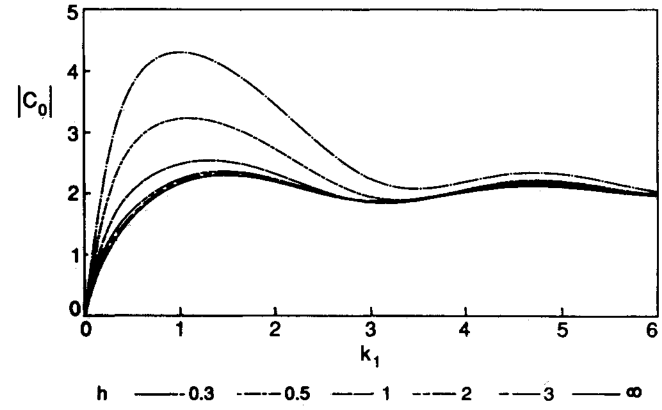

Fig. 4 Magnitude of the function C_0 vs k_1 at different wall distance h.

where

$$L_1'(k_1,h) = -\int_{-1}^{1} \Delta p(x')\,\mathrm{d}x' \tag{26}$$

Note that for $h \to \infty$, L_1' reduces to the Sears[8] function.

The total unsteady lift $L(h,t)$ acting on the airfoil and subject to the vortical disturbance (6) then is given by

$$L(h,t) = \int_{-\infty}^{\infty} \int_{0}^{\infty} \mathrm{X}(k_1,k_2,h,t) L_1'(k_1,h)\,\mathrm{d}k_2\,\mathrm{d}k_1 \tag{27}$$

III. Results and Discussion

We first note that, because of the splitting of the total unsteady velocity $\boldsymbol{u}$ into a vortical part $\boldsymbol{u}_\infty$ and an irrotational part $\boldsymbol{u}^{(1)}$, the coupling between them occurs only at the airfoil surface and its wake located at $x_2 = h$. This explains why the dependence on k_2 can be factored out by introducing X in Eqs. (18-20) and the resulting integral equation (17) does not depend on k_2. As a result, the unsteady lift L' depends on k_2 only through the known function X. Thus, for a given harmonic component (8), L' vanishes for $k_2 = n\pi/h$, where n is an integer. This corresponds to the cases in which the airfoil is located at an integer multiple of one-half the gust wavelength in the x_2 direction. On the other hand, $|\mathrm{X}|$ is maximum when $k_2 = (n + ½)\pi/h$, that is, when the airfoil is located at an odd integer multiple of one-quarter of the gust wavelength in the x_2 direction.

We now examine the variation of the other lift factor $L_1'(k_1,h)$ that must be obtained from the solution of the integral equation (17). As $k_1 \to \infty$, the integral of the right-hand side of Eq. (17) vanishes, and the contribution to the Cauchy integral comes mainly from the singularity of Δu at the leading edge. As a result, the limit of L_1' as $k_1 \to \infty$ is independent of h and identical to that of the Sears function; i.e.,

$$L_1' \to 1/\sqrt{2\pi k_1}\,\exp\{j(k_1 - \pi/4)\} \qquad \text{as } k_1 \to \infty \tag{28}$$

Similarly,

$$C_0 \to -2j\exp(-jk_1) \qquad \text{as } k_1 \to \infty \tag{29}$$

For $k_1 \to 0$, C_0 vanishes and Eq. (17) can be considered as the integral equation for the problem of a plate at a small angle of attack to a uniform flow in the presence of a wall. This problem was investigated by Widnall and Barrows,[3] and thus as $k_1 \to 0$, the values of L_1' tend to those of their lift. Figure 2 shows the variation of the magnitude of L_1' vs k_1 for various values of h, and Fig. 3 shows similar plots for the phase Φ^* of L_1' $\exp(-jk_1)$. At low reduced frequency, the effect of the wall is significant for $h < 2$. The magnitude of the unsteady lift almost doubles at $h = 0.3$. At higher k_1, the wall effect is significant only for $h < 0.25$; Φ^* does not practically depend on h for $k_1 < 0.5$. However, it decreases significantly for $k_1 > 0.5$ as h is reduced.

We now examine the effect of the wall on C_0. Figure 4 shows the variation of the magnitude of C_0 vs k_1 at various h, and Fig. 5 shows similar plots for the phase Ψ^* of C_0 $\exp(jk_1)$. For $k_1 = 0$, there is no vortex shedding in the wake and $C_0 = 0$. As for the lift, at low and moderate values of the reduced frequency, the presence of the wall enhances the magnitude of C_0 for $h < 2$. This result must be expected if we recall that C_0/k_1 is proportional to the unsteady circulation around the airfoil. The wall effect becomes less significant as k_1 is increased and $C_0 \exp(jk_1)$ tends to its asymptotic value of $-2j$. Note that both the magnitude and phase of C_0 are important in determining the unsteady velocity downstream of an airfoil because the dominant field is the one induced by the vortex sheet whose intensity at the trailing edge is C_0.

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Use of the Kirchhoff Method in Acoustics

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Introduction

SOUND can be considered as the propagating part of the full solution of some fluid-flow problems. In a few cases, the radiated sound can be found this way. However, in most cases of practical interest one cannot even numerically find a full solution everywhere in the flowfield because of diffusion and dispersion errors due to increasing mesh size in the far field. Thus, it is often advantageous, or even necessary, to develop ways of finding the far-field noise from near-field solutions.

The various approaches used to find the far-field noise can be classified as follows:

1) *Full Flowfield Solution:* Calculation of the full nonlinear flowfield including far-field waves. Practical grid densities are generally insufficient to numerically resolve the details of the acoustic three-dimensional far field.

2) *Acoustic Analogy:* Nonlinear near-field solution plus Ffowcs Williams and Hawkings equation. We should note that there are substantial difficulties in including the nonlinear quadrupole term in the volume integrals.

3) *Nonlinear Near-Field Plus Kirchhoff Method:* Calculation of the nonlinear near and midfield with the far-field solutions found from a linear Kirchhoff formulation evaluated on a surface surrounding the nonlinear field. The full nonlinear equations are solved in the first region (near field), usually numerically, and a surface integral of the solution over a control surface gives enough information for the analytical calculation in the second region (far field). The method has the advantage of including the full diffraction and focusing effects and eliminates the propagation of the reactive near field. This technique is the basis of this technical note.

Kirchhoff's formula was first published in 1882. Morgans[1] derived a Kirchhoff formula for a moving surface. Hawkings[2] and Morino[3,4] rederived that formula for some special cases. Recently, Farassat and Myers[5] rederived the general Kirchhoff formula using generalized derivatives. The authors also used the method[6,7] for calculation of far-field noise due to blade-vortex interactions (BVI).

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Formulation

A Green's function approach was used to derive the Kirchhoff formula in a coordinate system fixed to the airfoil which moves with a freestream velocity. Our approach is similar to that of Morino.[3,4] The Green's function approach is relatively simple and also the coordinate system used is suitable for BVI calculations. A full three-dimensional formulation is used, because the Green's function is simpler in this case, and because the method can be easily extended to include spanwise variations to model three-dimensional BVI.

If we assume that all of the acoustic sources are enclosed by an imaginary closed surface S, then we can prove using Green's theorem,[6,7] that the pressure distribution outside a rigid fixed surface $(\partial S/\partial t_1)$ is

$$p(x_o,y_o,z_o) = -\frac{1}{4\pi}\int_S \left[\frac{1}{r_o}\frac{\partial p}{\partial n_o} + \frac{1}{c_o r_o \beta^2}\frac{\partial p}{\partial t}\left(\frac{\partial r_o}{\partial n_o} - M\frac{\partial x_o'}{\partial n_o}\right) + \frac{p}{r_o^2}\frac{\partial r_o}{\partial n_o}\right]_\tau \mathrm{d}S_o' \tag{1}$$

where

$$r_o = \left\{(x-x')^2 + \beta^2\left[(y-y')^2 + (z-z')^2\right]\right\}^{1/2}$$

$$\tau = \frac{[r_o - M(x-x')]}{c_o\beta^2}$$

$$\beta = (1-M^2)^{1/2}$$

where " ′ " denotes a point on the Kirchhoff surface element $\mathrm{d}S_o'$ and subscript "o" denotes the transformed variables, using the well-known Prandtl-Glauert transformation:

$$x_o = x, \qquad y_o = \beta y \qquad z_o = \beta z$$

$n = (n_x, n_y, n_z)$ is the outward normal to the surface S, and subscript τ implies the evaluation at the retarded time $t_1 = t - \tau$.

Also using the Green's function approach, results can be obtained for a rotating Kirchhoff surface that might be useful for helicopter rotor calculations. The values of the pressure coefficient c_p and its normal derivatives on an arbitrary surface around an arbitrary flow are enough to give the far-field radiation at any arbitrary external point. In this work, we have chosen to use a rectangular box for the surface S.

Since Kirchhoff's method assumes that linear equations hold outside this control surface S, it must be chosen large enough to include the region of significant nonlinear behavior. However, due to increasing mesh spacing, the accuracy of the numerical solution is limited to the region immediately surrounding the moving blade. As a result, S cannot be so large as to lose accuracy in the numerical solution of the midfield. Thus, a judicious choice of S is required for the effectiveness of the Kirchhoff method in the case of nonlinear waves.

A half-period of a linear spherical sine wave is used as a test case for the method.

$$c_p(r,t) = \frac{\sin\left[\frac{(t-\tau)\pi}{A}\right]}{r_\beta} \qquad \text{for } 0 < t - \tau < A$$

$$c_p = 0 \qquad \text{elsewhere} \tag{2}$$

where A is the wave thickness (or half-period). The rectangular box-shaped control surface is shown in Fig. 1.

Results and Discussion

Figure 2 shows results for a freestream Mach number $M = 0.8$ at a point $(x, y, z) = (-0.5, 0, 0)$ for a reasonable rectangular-shaped control surface: $x_s = \pm 0.30$, $y_s = \pm 0.30$, (x_s, y_s are the coordinates of the control surface), 60×60 mesh points, mesh size $\Delta x = \Delta y = 0.01$, span = 4, 100 strips in the